This is a closed book exam. You are allowed ONE SIDE of ONE 8.5X11 formula sheet. There are 4 questions in this exam. The questions are worth 40, 15, 20 and 25 points. Please read each question very carefully. Make sure that you do the following:

1. Place your name on the front of every page of your exam.
2. Use only the FRONT of each page.
3. Draw a BOX around your final answer.
4. Provide extensive reasoning and mathematical justification for your answers to receive credit.

Good luck!
1. The average power-delay profile for a multipath channel is given in the figure above. Observe that in the figure power is given in linear scale on the y-axis, i.e., there is no need for conversion. The carrier frequency of the cellular system operating in this channel is 2GHz.

(a) (5 pts) What is the mean excess delay of this channel?

(b) (5 pts) What is the rms delay spread of this channel?

(c) (5 pts) What is the (ninety percent) coherence bandwidth of this channel?

(d) (10 points) Describe the type of fading experienced and how you would cope with it, if the symbol rate of the transmitter is

- 9.6 kbits/s
- 614.4 kbits/s

Assume the transmit signal bandwidth is inverse of bit duration. You must provide reasoning for your answers to receive credit.

(e) (15 points) Suppose that you are charged with designing the transmitter and the receiver for a wireless system that will operate in this channel. You are shooting for a very inexpensive design that does not allow for an equalizer on either side. On the other hand, you want the data rate to be at least 1Mbits/s. Propose a design that includes the modulation scheme that will not require an equalizer. You need to provide the block diagram and explain each step. Be as specific as possible with respect to numbers in the modulation scheme and the resulting bit rate. Assume, for the sake of simplicity, an uncoded system (i.e., ignore the error correction coding overhead).

\[
\text{a) mean excess delay: } \overline{\tau} = \frac{\sum_i p_i \tau_i}{\sum_i p_i} = \frac{1 \times 0 + 1 \times 1 + 0.25 \times 2 + 0 \times 3 + 0.25 \times 4}{1 + 1 + 0.25 + 0 + 0.25} = \frac{2.5}{2.5} = 1 \text{ ms}
\]
(b) \[ \text{rms delay spread} \quad \sigma_T = \sqrt{\bar{T}^2 - (\bar{T})^2} \]

\[ \bar{T}^2 = \frac{1 \times D^2 + 1 \times 1^2 + 0.25 \times 2^2 + D \times 3^2 + 0.25 \times 4^2}{1 + 1 + 0.25 + 0.25} = \frac{1+1+4}{2.5} = 2.4 \text{ (\mu s)}^2 \]

From (a) \( \bar{T} = 1 \text{ \mu s} \)

So \( \sigma_T = \sqrt{2.4 - 1^2} = \sqrt{1.4} \approx 1.2 \text{ \mu s} \)

(c) \[ B_c \approx \frac{1}{50 \sigma_T} = \frac{1}{50 \times 1.2 \times 10^{-6}} \text{ Hz} = \frac{100 \times 10^3 \text{ Hz}}{6} = 16.67 \text{ kHz} \]

(d)

- 9.6 kbits/s

\[ \text{BW} \approx 9.6 \text{ kHz} < 16.67 \text{ kHz} = B_c \]

⇒ Flat fading (Signal fits in the coherence bandwidth of the channel)

- 614.4 kbits/s

\[ \text{BW} \times 614.4 \text{ kHz} > 16.67 \text{ kHz} \]

⇒ Frequency selective fading (Signal BW larger than the coherence BW of the channel)

(2) You need to use channels of at most 16.7 kHz bandwidth in order not to have to employ equalizers. Since you need data rate to be 1 Mbit/s, you need 60 such channels and hence multicarrier transmission.

A 64 subcarrier OFDM system is a good candidate where binary modulation is sufficient.

Alternatively, we can go for fewer tones but use a modulation scheme with more than 1 bit/symbol.

Example: 4 bits/symbol - 16 QAM and 16 subcarriers. See notes for block diagram.
2. Assume a mobile using the cellular system in question 1, i.e. the system with 2GHz carrier frequency. The mobile’s speed can go up to 100 miles/hr.

(a) (5 pts) What is the maximum doppler shift that the signal between this mobile and the base station would experience?

\[ f_{\text{max}} = \frac{V}{\lambda} = \frac{400}{9} \frac{1}{0.15} = \frac{40000}{135} = \frac{8000}{27} \approx 300 \text{ Hz} \]

\[ V = \frac{100 \text{ miles}}{\text{hr}} = \frac{100 \times 1600 \text{ m}}{3600 \text{ sec}} = \frac{400}{9} \text{ m/s} \]

\[ \lambda = \frac{1}{fc} = \frac{3 \times 10^8 \text{m/sec}}{2 \times 10^9 \text{v/sec}} = 0.15 \text{m} \]

(b) (10 pts) What range of transmitted symbol rates would go through this channel without time-selectivity (in other words, what is the longest symbol duration that would experience slow fading in this channel)? You need to provide details and explanation in each step.

\[ T_c = \frac{1}{2f_m} = \frac{1}{600} \text{ sec} \quad \text{channel coherence time} \]

So if symbol duration is less than \( T_c \) above, each symbol will experience more than one channel realization, i.e. fast fading.

That is to say, the symbol duration must be less than \( \frac{1}{600} \text{ sec} = 1.67 \text{ msec} \)

(In practice, it should be \( \ll \frac{1}{600} \text{ sec} \) so that blocks of symbols experience slow fading.)
3. (20 points) Suppose that you are charged with designing a wireless system. Specifically, you are asked to find out what transmit power will be needed at the base station tower. You first go and get extensive measurements on the propagation environment and find that a power law with $n = 3$ is a good model for the median pathloss. You also find that all losses due to cable, antenna etc. is modeled well with a fixed $10\,dB$ loss. Assuming a cell phone receiver sensitivity of $-90\,dBm$, what transmitter power is required at the base station to service a circular area of radius $15$ km? Now assume that, the standard deviation about the median path loss was estimated to be $8\,dB$. Assuming a lognormal shadowing model, how much additional power must be transmitted to cover the same service area with $90\%$ availability at the edge of coverage when shadowing is taken into account? (A Q-fct table is attached for your convenience).

\[ n = 3 \]
\[ P_{\text{min}} = -90 \, \text{dBm} = -120 \, \text{dBW} \]
\[ P_T - 10 \, \text{dB} - 10 \log r^n \geq P_{\text{min}} \]
\[ P_T - 10 \, \text{dB} - 30 \log (15 \times 10^3) \geq -120 \, \text{dBW} \]
\[ P_T - 30 \log 15 - 90 \geq -110 \]
\[ P_T \geq 30 \log 15 - 20 = 15.28 \, \text{dBW} \]
\[ P_T = 15.28 \, \text{dBW} = 33.75 \, \text{W} \]

\[ \sigma = 8 \, \text{dB} \quad \text{shadowing} \quad , \quad 90\% \text{ availability} \]

i.e. we want

\[ P \left[ P_T - L_p(r) - S > -120 \right] = 0.9 \]

or equivalently

\[ P \left[ P_T - S < 15.28 \right] = 0.1 \]

\[ P \left[ S > P_T - 15.28 \right] = 0.1 \]

Recall $S \sim N(0, 64)$

\[ \int_{-\infty}^{S} \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{\alpha^2}{2\sigma^2}} \, d\alpha = \Phi \left( \frac{S}{\sigma} \right) \]

\[ \Phi (x) = 0.1 \quad \Rightarrow \quad x = 1.3 \quad \text{From table} \]

\[ \frac{S}{\sigma} = 1.3 \quad \Rightarrow \quad S = 10.4 \, \text{dB} \quad \text{additional power needed} \]
4. This question is on modulation and multiple access methods.

(a) (6 pts) In class, we have seen pros and cons of different digital modulation schemes. Pick one of the following two and list the pros and cons explaining your reasoning in detail (i) FSK vs PSK; (ii) FSK vs ASK.

(b) (10 pts) A wireless system needs to be designed to accommodate both voice and real-time data traffic. The real-time data (e.g. streaming video) requires roughly ten times the data rate of voice. You have a choice of using any multiple access scheme in the design. Propose a system design justifying your choice of the multiple access scheme to use to share the resources between the users as well as the two types of data. You can use an existing system (the examples we have seen in class) or parts of it if you like.

(c) (9 pts) For this system you are designing, you need to also choose a modulation scheme. You are given that voice traffic requires a bit error rate (BER) of $10^{-3}$ or smaller. Data traffic is more demanding and requires $10^{-5}$. Explain what type of modulation scheme you would use (you can use any binary or quaternary scheme) and why. Provide the $E_b/N_0$ values you would need for voice and data traffic with your choice of the modulation scheme (A Q-fct table is attached for your convenience).

(a) (i) PSK is bandwidth efficient. FSK is not.
PSK suffers from higher error for large constellations. FSK provides better performance at the expense of higher bw.
FSK enables non-coherent detection. PSK must have phase info and co. detection.

(ii) FSK power efficient, BW inefficient
FSK 

(b) Use a TDMA based system. Each frame has $K$ time slots. Nominal data rate is when each user transmits once in a frame. For users with higher data rate demands, i.e. $k > 1$, assign $k$ slots to the user.

(c) $E_b/N_0$ for voice

$$Q(\frac{\sqrt{2E_b}}{N_0}) = 10^{-3} \Rightarrow x = 3.1$$

$$E_b/N_0 = \frac{(3.1)^2}{2} = 4.81$$

$E_b/N_0$ for data

$$Q(\frac{\sqrt{2E_b}}{N_0}) = 10^{-5} \Rightarrow \frac{E_b}{N_0} = \frac{(4.3)^2}{2} = 9.25$$

Use QPSK