EE497 A SP 2010
HW 4 Solutions

1) The system is in outage if the output SNR < 10

a) Avg. branch SNR is 10 dB = 10 and is identical for all
antennas. $P_{out} = P(SNR < 10) = (1 - e^{-10/10})^{N_r}$

$N_r = 1$ (No diversity) $P_{out} = P(SNR < 10) = (1 - e^{-1}) = 0.63$

$N_r = 2$ $P_{out} = ((1 - e^{-1})^2 = 0.40$

$N_r = 3$ $P_{out} = (1 - e^{-1})^3 = 0.25$

b) $N_r = 3$ $\delta_{Avg1} = 10 dB = 10$

$\delta_{Avg2} = 15 dB = 10^{1.5} = 31.63$

$\delta_{Avg2} = 20 dB = 100$

$$P_{out} = P(max(\delta_1, \delta_2, \delta_3) < 10) = P(\delta_1 < 10, \delta_2 < 10, \delta_3 < 10)$$

$$= P(\delta_1 < 10) \cdot P(\delta_2 < 10) \cdot P(\delta_3 < 10)$$

$\downarrow$ indep. of antennas

$$P(\delta_i < 10) = 1 - e^{-10/\delta_{Avgi}} \quad i = 1, 2, 3$$

Thus

$$P_{out} = (1 - e^{-10/10}) \cdot (1 - e^{-10/31.63}) \cdot (1 - e^{-10/100})$$

$$= 0.63 \times 0.27 \times 0.095 = 0.016$$

Note the significant performance difference:

With 3 antennas each of which has 10 dB SNR,
the outage prob. is 25%.

With 3 antennas two of which have 5 + 10 dB
additional gain, the outage prob. reduced to 1.6%
2. This was derived on the board in class in sufficient details. You needed to give all the steps (not just the summary) to receive credit.

3. \[ 5 \text{ MHz total BW} \rightarrow T_c \approx 0.2 \times 10^{-6} \text{ s} \]
\[ 10 \text{ kbps data rate} \rightarrow \tau_b \approx 10^{-4} \text{ s} \]
\[ N = \frac{\tau_b}{T_c} = \frac{10^{-4}}{0.2 \times 10^{-6}} = 500 \quad \text{Prosenary gain} \]
\[ \gamma^* = 7 \text{ dB} = 5 \]

a) With perfect P.C. and random signatures, we have
\[ \gamma = \frac{p}{(K-1)p + \sigma^2} = \gamma^* \quad \Rightarrow \quad p = \frac{\gamma^* (K-1)p + \sigma^2}{(1-\gamma^*(K-1))p} = \frac{\sigma^2}{1-(\gamma^*(K-1)p)} \]
\[ \frac{1}{(K-1)} \gamma^* < 1 \]
\[ K < \frac{N}{\gamma^*} + 1 = \frac{500}{5} + 1 = 101 \]
Up to (and including) 100 users can be supported.

b) If we have decorrelating receivers then
\[ \tilde{y} = R^{-1} y \quad (y = \text{RAE} + \zeta) \]
and
\[ \hat{y}_i = A_i e_i + \tilde{n}_i \quad \zeta \sim N(0, \sigma^2) \]
and
\[ \hat{e}_i = \text{sign} (\tilde{y}_i) \]
Given that we can suppress the interference completely, the "SIR" (really SNR) at the output of a decorrelator is simply

\[ \delta_i = \frac{P_i}{\sigma^2 (R^{-1})_{ii}} \]

Thus, without a maximum power constraint we should be able to support as many users as possible PROVIDED THAT THE DECORRELATING DETECTOR EXISTS!

This means that we can support users as long as the corresponding cross correlation matrix \( R \) is INVERTIBLE.

Note

\[ R_{ij} = R_{ij} = S_i^T S_j \]

Thus \( R \) can be expressed as \( R = S^T S \)

with \( S = \begin{bmatrix} S_1 & S_2 & \cdots & S_K \end{bmatrix} \) \( N \times K \)

\[ \text{rank}(S) = \min(N,K) \]

\[ \text{rank}(AB) \leq \text{rank}(A) \Rightarrow \text{rank}(R) = \text{rank}(S^T S) \leq \min(N,K) \]

\( R \) is a \( K \times K \) matrix, so it can not be invertible (i.e. it is rank deficient) when \( K > N \)

\[ \Rightarrow \text{Thus, we can support up to (and including)} \]

\[ K = N = 500 \text{ users with decorrelating receivers.} \]